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(SAADO) for a 3-D Flexible Wing**

Clyde R. Gumbert
NASA Langley Research Center
Hampton, Va 23681-2199

Gene J. -W. Hou
Old Dominion University
Norfolk, VA 23529 - 0247

Perry A. Newman
NASA Langley Research Center
Hampton, VA 23681-2199

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Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) for a 3-D Flexible Wing

Clyde R. Gumbert^{*}, Gene J.-W. Hou[†], Perry A. Newman[‡]

ABSTRACT

The formulation and implementation of an optimization method called Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) are extended from single discipline analysis (aerodynamics only) to multidisciplinary analysis – in this case, static aero-structural analysis – and applied to a simple 3-D wing problem. The method aims to reduce the computational expense incurred in performing shape optimization using state-of-the-art Computational Fluid Dynamics (CFD) flow analysis, Finite Element Method (FEM) structural analysis and sensitivity analysis tools. Results for this small problem show that the method reaches the same local optimum as conventional optimization. However, unlike its application to the rigid wing (single discipline analysis), the method, as implemented here, may not show significant reduction in the computational cost. Similar reductions were seen in the two-design-variable (DV) problem results but not in the 8-DV results given here.

NOMENCLATURE

b	wing semispan
C_D	drag coefficient
C_l	rolling moment coefficient
C_L	lift coefficient
C_m	pitching moment coefficient
C_p	pressure coefficient
c_r	wing root chord

c_t	wing tip chord
F	design objective function
g	design constraints
K	stiffness matrix
L	aerodynamic loads
M_∞	free-stream Mach number
P	compliance, the work done by the aerodynamic load to deflect the structure
q_∞	free-stream dynamic pressure
Q	flow-field variables (state variables) at each CFD mesh point
ΔQ_1	change in flow solver field variables due to better analysis convergence
ΔQ_2	change in flow solver field variables due to design changes
R	state equation residuals at each CFD mesh point
$ R/R_0 $	norm of the residual ratio, current/initial
S	semispan wing planform area
u	structural deflections
Δu_1	change in deflections due to better analysis convergence
Δu_2	change in deflections due to design changes
W	wing weight
X	CFD mesh coordinates
x_{LE}	vector location of wing root leading edge
x/c	chordwise location normalized by local wing section chord
x_t	longitudinal location of wing tip trailing edge
z_r	root section camber parameter
α	free-stream angle-of-attack

^{*} Research Scientist, Multidisciplinary Optimization Branch, M/S 159, NASA Langley Research Center, Hampton, VA 23681-2199, c.r.gumbert@larc.nasa.gov.

[†] Professor, Department of Mechanical Engineering, Old Dominion University, Norfolk, VA 23529-0247, ghou@lions.odu.edu, Member AIAA.

[‡] Senior Research Scientist, M/S 159, NASA Langley Research Center, Hampton, VA 23681-2199, p.a.newman@larc.nasa.gov.

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β	design variables
Δ	operator which indicates a change in a variable
ϵ, ϵ'	convergence tolerances
γ	line search parameter
τ	twist angle at wing tip, positive for leading edge up
subscripts:	
b	baseline volume mesh
d	deflected shape
j	jig (undeflected) shape
s	wing surface
v	volume
superscripts:	
*	designates updated value
'	gradient with respect to design variables

INTRODUCTION

Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) is a procedure that incorporates design improvement within the iteratively solved (nonlinear) aerodynamic analysis so as to achieve fully converged flow solutions only near an optimal design. When SAADO is applied to a flexible wing rather than a rigid wing, the linear FEM solution is iteratively coupled with the nonlinear CFD solution. Overall computational efficiency is achieved because the many expensive iterative (nonlinear) solutions for non-optimal design parameters are not converged (i.e., obtained) at each optimization step. One can obtain the design in the equivalent of a few (rather than many) multiples of computational time for a single, fully converged coupled aero-structural analysis. SAADO and similar procedures for simultaneous analysis and design (SAND) developed by others are noted and discussed by Newman et al.¹ These SAND procedures appear best suited for applications where discipline analyses involved in the design are nonlinear and solved iteratively. Generally, convergence of these discipline analyses (i.e., state equations) is viewed as an equality constraint in an optimization problem. From this latter point of view, the SAADO method proceeds through infeasible regions of the (β , Q , u) design space. A further advantage of SAADO is the efficient use of existing discipline analysis codes (without internal changes), augmented with sensitivity or gradient information, and yet effectively coupled more tightly than is done in conventional gradient-based optimization procedures, referred to as nested analysis and design (NAND) procedures.¹ A recent overview of aerodynamic shape optimization² discusses both NAND and SAND procedures in the context of current steady aerodynamic optimization research.

For single-discipline design problems, the distinction between NAND and SAND procedures is fairly clear and readily seen. With respect to discipline feasibility (i.e., convergence of the generally nonlinear, iteratively solved state equations), these procedures can be viewed as accomplishing design by using only well converged discipline solutions (NAND) or as a sequence of discipline solutions converged from poorly to well as the design progresses (SAND). However, the problem formulation and solution algorithms may differ considerably. About twenty SAND references are quoted by Newman et al.¹ and Newman et al.²; these references discuss a variety of formulations, algorithms, and results for single-discipline problems (mostly CFD applications) in the sense of SAND defined above. For multidisciplinary design optimization problems, the distinction between NAND and SAND is somewhat blurred because there are feasibility considerations with respect to all individual discipline state equations as well as with respect to multidisciplinary system compatibility and constraints. A number of the papers in Ref. 3 discuss MDO formulations and algorithms that are called SAND-like. However, not all of these latter MDO procedures appear to agree with the sense of SAND defined above and used here; one that does is Ref. 4.

The computational feasibility of SAADO for quasi 1-D nozzle shape design based on the Euler equation CFD approximation was demonstrated by Hou et al.⁵ and Mani.⁶ Application of SAADO for turbulent transonic airfoil shape design based on a 2-D thin-layer Navier-Stokes CFD approximation was demonstrated and reported in a later paper by Hou et al.⁷ Both of these application results are summarized and briefly discussed in Ref. 1. The application of SAADO for rigid 3-D wing design based on the Euler CFD approximation was presented in Ref 8. These SAADO procedures utilized quasi-analytical sensitivity derivatives obtained from hand-differentiated code for the initial quasi 1-D application,^{5,6} and from automatically differentiated code for both the 2-D airfoil application⁷ and the 3-D rigid wing application.⁸ Different optimization techniques have also been used in these SAADO procedures.

The flexible wing studied here is formulated as a static aeroelastic problem. Similar problems have been used as examples in Refs. 9–14 to study various solution strategies for multidisciplinary analysis and optimization. In particular, Arian¹¹ analyzed the Hessian matrix for the system equations to derive mathematical conditions under which the aeroelastic optimization

problem can be solved in a "loosely" coupled manner. Multidisciplinary research of Walsh et al.^{12,13} emphasized the engineering aspects of integrating high fidelity disciplinary analysis software and distributed computing over a network of heterogeneous computers. The aeroelastic analysis results of Reuther et al.¹⁴ were verified with experimental data.

Only a limited amount of literature related to aeroelastic problems has elaborated on the coupled sensitivity analysis. Kapania, Eldred and Barthelemy¹⁵; Arslan and Carlson¹⁶; and Giunta and Sobieszczanski-Sobieski¹⁷ derived global sensitivity equations (GSEs), with some matrix coefficients in these GSEs evaluated by finite differencing. Guinta¹⁸ later introduced modal coordinates to approximate the elastic displacement vector in order to reduce the size of the GSE. Newman, Whitfield, and Anderson¹⁹ used the complex variable approach to obtain aeroelastic sensitivity derivatives, whereas Reuther et al.¹⁴ employed the adjoint variable approach to derive aeroelastic sensitivity equations. A mathematical study of the coupled nonlinear, incompressible aeroelastic analysis and sensitivity analysis problems was performed by Ghattas and Li.²⁰ Recent results on aeroelastic sensitivity analysis and optimization can be found in Refs. 21-23. Particularly, Farhat²² and Hou and Satyanarayana²³ explicitly formulated deflection updates and load transfers between the separate flow and structures solvers as part of the coupled sensitivity equations. In the present study, coupled sensitivity equations are constructed by differentiating the aeroelastic state equations and solving them by a Generalized Gauss-Seidel (GGS) method.¹¹ The present SAADO concept is very similar to that of Ghattas and others, Refs. 4, 20, 24, and 25, but differs in derivation and implementation details as described later.

Our initial 3-D flexible wing results from SAADO are given in this paper. The problem is the same simple wing planform as used in Ref. 8 for rigid wing design studies. Here, changes in design variables are sought to produce improvement in the lift-to-drag ratio subject to both aerodynamic and structural solution-dependent constraints. These constraints are the difference between the lift and weight, the pitching moment coefficient, and the compliance. The latter is a function representing work done by the aerodynamic load to deflect the structure. There are also geometric constraints. Note that the structural interaction at both the function (analysis) and derivative (sensitivity analysis) levels must be included even when only aerodynamic design variables are involved. The ultimate goal of our work is to extend the SAADO procedure to flexible wing design problems that also

involve structural design variables and additional structural responses in the problem formulation.

PROBLEM DESCRIPTION

To evaluate efficacy of the SAADO procedure for a problem involving multidisciplinary analysis, it is applied herein to a simple, isolated, flexible wing. The wing consisted of a trapezoidal planform with a rounded tip. It was parameterized by fifteen variables; five described the planform, and five each described the root and tip section shapes. A schematic of the wing and its associated planform parameters is shown in Fig. 1. The baseline wing section varied linearly from an NACA 0012 at the root to an NACA 0008 at the tip. The specific parameters selected as design variables in sample optimization problems are identified in the section entitled Results. The objective function to be minimized was the negative of the lift-to-drag ratio, $-L/D$. Both coupled solution-dependent and geometric constraints were imposed.

The solution-dependent constraints were:

- lower limit on the difference between total lift and structural weight, $(C_L * S * q_\infty - W)$ ($W = \text{constant}$ for rigid problem)
- upper limit on compliance, P (for flexible wing problem)
- upper limit on rolling moment coefficient, C_r , in lieu of bending moment limits (for rigid wing problem)
- upper limit on pitching moment, C_m , in lieu of a trim constraint

The purely geometric constraints were:

- minimum leading edge radius, in lieu of a manufacturing requirement
- side constraints (bounds) on active design variables

SAADO PROCEDURE

Formulation

The flexible SAADO approach formulates the design-optimization problem as follows:

$$\min_{\beta, Q, u} F(Q, X_{dv}(\beta, u), u, \beta) \quad (1)$$

subject to

$$g_i(Q, X_{dv}(\beta, u), u, \beta) \leq 0; \quad i = 1, 2, \dots, m \quad (2)$$

where flow field variables Q and structural deflections u are a solution of the coupled flow equation

$$R(Q, X_{dv}(\beta, u), \beta) = 0 \quad (3)$$

and finite element structural equation

$$K(X_j(\beta))u = L(Q, X_{dv}(\beta, u)) \quad (4)$$

The deflected volume mesh, X_{dv} , is determined by the deflected surface mesh, X_{ds} , as $X_{dv} = X_{dv}(X_{ds}, X_{bv})$. This deflected surface mesh is a result of the jig shape augmented by elastic deflections, u , as $X_{ds} = X_j(\beta) + u$. The two disciplines are coupled through deflections, u , and loads, L .

Recall that Q , R , and X_{dv} are very large vectors. This formulation treats the state variables, Q and u , as part of the set of independent design variables, and considers the state equations as constraints. Because satisfaction of the equality constraints, Eqs. (3) and (4), is required only at the final optimum solution, coupled steady-state aero-structural field equations are not converged at every design-optimization iteration. Easing of this requirement is expected to significantly reduce excessively large computational costs incurred in the conventional approach. However, this advantage would most likely be offset by the very large increase in the number of design variables and equality constraint functions, unless some remedial procedure is adopted.

Approximations

The SAADO method begins with a linearized design-optimization problem which is solved for the most favorable change in design variables, $\Delta\beta$, as well as for changes in state variables, ΔQ and Δu ; that is,

$$\begin{aligned} \min_{\Delta\beta, \Delta Q, \Delta u} F(Q, X, \beta) \\ + \frac{\partial F}{\partial Q} \Delta Q + \frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u \\ + \left(\frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial F}{\partial \beta} \right) \Delta\beta \end{aligned} \quad (5)$$

subject to inequality constraints

$$\begin{aligned} g_i(Q, X, \beta) + \frac{\partial g_i}{\partial Q} \Delta Q + \frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u \\ + \left(\frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial g_i}{\partial \beta} \right) \Delta\beta \leq 0; \quad i = 1, 2, \dots, m \end{aligned} \quad (6)$$

and equality constraints

$$\begin{aligned} R(Q, X, \beta) + \frac{\partial R}{\partial Q} \Delta Q + \frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u \\ + \left(\frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial R}{\partial \beta} \right) \Delta\beta = 0 \end{aligned} \quad (7)$$

and

$$\begin{aligned} K(X_j(\beta))u - L(Q, X_{dv}) + \left(K - \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) \Delta u \\ - \frac{\partial L}{\partial Q} \Delta Q + \left(\frac{\partial K}{\partial X_j} u - \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) X'_j \Delta\beta = 0 \end{aligned} \quad (8)$$

Note that Eqs. (5) through (8) are linearized approximations of Eqs. (1) through (4), respectively.

In this formulation, neither the residual of the non-linear aerodynamic field equations, $R(Q, X, \beta)$, nor that of the structures equation, $Ku - L$, is required to be zero (reach target) until the final optimum design is achieved. The linearized problem of Eqs. (5) through (8) is difficult to solve directly because of the number of design variables and equality constraint equations. This difficulty is overcome for the direct differentiation method by using direct substitution to remove ΔQ , Δu , and Eqs. (7) and (8) altogether from this linearized problem; that is, one expresses ΔQ and Δu as functions of $\Delta\beta$.

$$\begin{aligned} \Delta Q &= \Delta Q_1 + \Delta Q_2 \Delta\beta \\ \Delta u &= \Delta u_1 + \Delta u_2 \Delta\beta \end{aligned} \quad (9)$$

where vectors ΔQ_1 and Δu_1 and matrices ΔQ_2 and Δu_2 are solutions of the following coupled sets of equations, obtained from Eqs. 7 and 8,

$$\begin{aligned} \frac{\partial R}{\partial Q} \Delta Q_1 + \frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u_1 &= -R \\ K \Delta u_1 &= \frac{\partial L}{\partial Q} \Delta Q_1 + \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u_1 \end{aligned} \quad (10)$$

where, for the linear FEM, $Ku - L = 0$ at every iteration, and

$$\begin{aligned} \frac{\partial R}{\partial Q} \Delta Q_2 + \frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} (X'_j + \Delta u_2) + \frac{\partial R}{\partial \beta} &= 0 \\ K \Delta u_2 &= \frac{\partial L}{\partial Q} \Delta Q_2 + \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} (X'_j + \Delta u_2) \\ - \frac{\partial K}{\partial X_j} X'_j u & \end{aligned} \quad (11)$$

Note that the number of columns of matrices ΔQ_2 and Δu_2 is equal to the number of design variables, β ; thus

the computational cost of Eq. (11) is directly proportional to the number of design variables.

A new linearized problem with $\Delta\beta$ as the only design variables can be obtained by substituting Eq. (9) into Eqs. (5) and (6) for ΔQ and Δu :

$$\min_{\Delta\beta} F(Q, X, \beta) + \frac{\partial F}{\partial Q} \Delta Q_1 + \frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u_1 + \left(\frac{\partial F}{\partial Q} \Delta Q_2 + \frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} (X'_j + \Delta u_2) + \frac{\partial F}{\partial \beta} \right) \Delta\beta \quad (12)$$

subject to

$$g_i(Q, X, \beta) + \frac{\partial g_i}{\partial Q} \Delta Q_1 + \frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u_1 + \left(\frac{\partial g_i}{\partial Q} \Delta Q_2 + \frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} (X'_j + \Delta u_2) + \frac{\partial g_i}{\partial \beta} \right) \Delta\beta \leq 0, \quad (13)$$

$i=1,2,\dots,m$

Note that the expressions inside large parentheses in Eqs. (12) and (13) are approximated gradients of the objective and constraint functions. Once established, this linearized problem can be solved using any mathematical programming technique for design changes, $\Delta\beta$.

Line Search

A one-dimensional search on the step size parameter γ is then performed in order to find updated values of β^* , X^* , Q^* , and u^* . This line search functions to adjust the magnitude of $\Delta\beta$ so as to simultaneously ensure better results for both design and analysis (converged solutions). The step size parameter γ plays the role of a relaxation factor in the standard Newton's iteration. The search procedure employed solves a nonlinear optimization problem of the form

$$\min_{\gamma} F(Q^*, X^*, \beta^*) \quad (14)$$

subject to

$$g_i(Q^*, X^*, \beta^*, u^*) \leq 0; \quad i=1,2,\dots,m \quad (15)$$

$$R(Q^*, X^*, \beta^*) = 0 \quad (16)$$

and

$$K(X^*)u^* = L(Q^*, X^*, \beta^*) \quad (17)$$

where step size γ is the only design variable. Again it is noted for emphasis that equality constraints, Eqs. (16) and (17), are not required to be zero (reach target) until the final optimum design; violations of

these equality constraints must simply be progressively reduced until the SAADO procedure converges.

The updated Q^* and u^* can be viewed as $Q^* = Q + \Delta Q^*$ and $u^* = u + \Delta u^*$ where ΔQ^* and Δu^* satisfy the first order approximations to Eqs. (16) and (17). That is, ΔQ^* and Δu^* are the solutions of Eqs. (7) and (8) where, in Eq. (9), $\Delta\beta$ is replaced by $\Delta\beta^* = \gamma\Delta\beta$. Consequently, $Q^* = Q + \Delta Q_1 + \gamma\Delta Q_2\Delta\beta$ and

$u^* = u + \Delta u_1 + \gamma\Delta u_2\Delta\beta$ are readily available once γ is

found. The $\Delta(\cdot)_1$ terms appearing in the above SAADO formulation are due to better convergence of the coupled analysis, whereas $\Delta(\cdot)_2$ terms are due to changes in design variables. In fact, ΔQ_2 and Δu_2 approach the flow field and deflection sensitivities, Q' and u' , as the solution becomes better converged. The appearance of ΔQ_1 and Δu_1 in the formulation makes the SAADO approach different from the conventional NAND aerodynamic optimization method. The ΔQ_1 and Δu_1 not only constitute changes in Q and u , but also play important roles in defining the constraint violation of Eq. (13). Since ΔQ_1 and Δu_1 , as shown in Eq. (10), represent a single Newton's iteration on the coupled equations, it is possible to approximate them as the changes in Q and u as a result of several Newton's iterations to improve quality of the solution as was done in this study.

Implementation

The following pseudocode shows algorithmically how the method was implemented.

```

set initial analysis convergence tolerance,  $\epsilon$ 
set initial solution vectors,  $Q$  and  $u$ 
set initial design variables,  $\beta$ 
do until converged
    1. solve Eqs. (3) & (4) partially converged to  $\epsilon$ 
    2. compute  $F$  and  $g$ 
    3. solve Eq. (11) partially converged to  $\epsilon'$ 
    4. compute  $\Delta\beta$  terms of Eqs (12) & (13)
    5. solve optimization problem Eq (12) & (13) for  $\Delta\beta$ 
    6. solve Eqs (14) - (17) for line search parameter,  $\gamma$ 
    7. update  $\beta$ ,  $u$ , and  $Q$ 
    8. tighten analysis convergence tolerance,  $\epsilon = \epsilon * \text{factor}$ ,  $\text{factor} < 1$ 
enddo

```

This pseudocode is similar to that used in the Biros and Ghattas²⁴ SAND approach. Specifically, both

approaches use an SQP method to solve the design equation (step 5) and an approximate factorization method to solve the system equations (step 1). Step 3 above uses an incremental iterative method with approximate factorization to solve for derivatives in direct mode rather than as a solution of the adjoint equation of Biros and Ghattas.²⁴ In addition, the line search step (step 6) and the convergence tightening step (step 8) were not included in the Biros and Ghattas method.

A schematic of the present SAADO procedure is shown in Fig. 2. The dashed box, labeled "Partially Converged System Analysis," depicts the coupled analysis iteration loop, Steps 1 & 2 of the pseudocode; that labeled "Partially Converged Sensitivity Analysis" depicts the coupled derivative iteration loop, Step 3; and that labeled "Partially Converged Design" depicts the design steps, Steps 5 - 8 of the pseudocode. Specific computational tools and methods used to perform the tasks depicted by the solid boxes in Fig. 2 are identified in the next section.

COMPUTATIONAL TOOLS AND MODELS

Major computations in this SAADO procedure are performed using a collection of existing codes. These codes are executed by a separate driver code and scripts that implement the SAADO procedure as just discussed. Each code runs independently, perhaps simultaneously, on different processors, and the required I/O transfers between them, also directed by the driver code, are accomplished by data files.

The aerodynamic flow analysis code used for this study is a version of the CFL3D code.²⁶ Only Euler analyses are performed for this work, although the code is capable of solving Navier-Stokes equations with any of several turbulence models. The derivative version of this code, which was used for aerodynamic sensitivity analysis, was generated by an unconventional application²⁷ of the automatic differentiation code ADIFOR^{28,29} to produce a relatively efficient, direct mode, gradient analysis code, CFL3D.ADII.³⁰ It should be pointed out that the ADIFOR process produces a discretized derivative code that is consistent with the discretized function analysis code. Addition of a stopping criterion based on the norm of the residual of the field equations was the only modification made to the CFL3D.ADII code to accommodate the SAADO procedure.

Surface geometry was generated based on parameters described in a previous section by a code utilizing the Rapid Aircraft Parameterization Input Design (RAPID) technique developed by Smith, et al.³¹ This

code was preprocessed with ADIFOR to generate a code capable of producing sensitivity derivatives, X_j' , as well.

The CFD volume mesh needed by the flow analysis code was generated using a version of the CSCMDO³² grid generation code. Associated grid sensitivity derivatives needed by the flow sensitivity analysis were generated with an automatically differentiated version of CSCMDO.³³ In addition to the parameterized surface mesh and accompanying gradients, CSCMDO requires a baseline volume mesh of similar shape and identical topology. The 45,000 grid point baseline volume mesh of C-O topology used in the present flexible wing examples was obtained with the GridgenTM code. The 41,000-point baseline volume mesh used in the rigid wing optimization problem was generated using WTCO³⁴. These meshes are admittedly particularly coarse by current CFD analysis standards; the wing surface meshes are shown in Fig. 3.

The structural analysis code³⁵ used to compute the deflection of the elastic wing was a generic finite element code. The flexible structure for the wing shown in Fig. 3 was discretized by 583 nodes; there were 2141 constant-strain triangle (CST) elements and 1110 truss elements. Because the elastic deformation was assumed to be small, linear elasticity was deemed appropriate. The structural sensitivity equation was derived based upon the direct differentiation method. Note that sensitivity of the aerodynamic forces appears as a term on the right-hand side (RHS) of the deflection sensitivity equation. The derivative of the stiffness matrix in this sensitivity equation was also generated³⁶ by using the ADIFOR^{28,29} technique. Since the coefficient matrix of the structural sensitivity equation was identical to that of the structural equation, these sensitivity equations were solved efficiently by backward substitution with different RHSs for each sensitivity.

At the wing surface, i.e., the interface where aerodynamic load and structural deflection information is transferred, it was assumed that surface nodes of the Finite Element Method (FEM) structural model were a subset of CFD aerodynamic surface mesh points (see Fig. 3) for the present SAADO application. This lack of generality allowed for simplifications in data transfers and, although an important issue, it was not deemed crucial for these initial flexible wing SAADO demonstrations. Future applications to more complex configurations should allow for transfer of conserved information between

arbitrary meshes as required by individual disciplines. A recent review of such data transfer techniques and specific recommendations are given in Ref. 37.

Conventional (NAND) and SAADO (SAND) procedures were implemented using the Sequential Quadratic Programming method of the DOT³⁸ optimization software. All computations were executed on an SGI Origin 2000™ computer with 250Mhz R10000™ processors. The CFD sensitivity calculations were partitioned and run on several processors to reduce required memory and elapsed optimization time. This partitioning, however, results in additional accumulated computational time due to the nature of ADIFOR-generated sensitivity analysis code.

RESULTS

Figures 4 and 5 show the effect on convergence and computational cost of coupling the CFD and FEM analysis and sensitivity solvers, respectively. The mesh or mesh derivatives are updated with the deflections or deflection derivatives, respectively, as indicated by the symbols. Even with a relatively flexible wing, there is little effect on the convergence rate, i.e., residual reduction per CFD iteration. However, the computation (cpu) time does increase – rather dramatically for the coupled function analysis in Fig. 4 – due to repeated input and output of large mesh and restart files in the CFD flow solver and frequent mesh regeneration. The cpu time spent performing the FEM calculations and the interface of coupling data are too small to be visible in this figure.

The cpu time shown in Fig 5 is the cost for sensitivity analysis for two design variables. For clarity only one convergence history was shown; the other was nearly identical. The cost of the function analysis relative to the sensitivity analysis is greater than that anticipated from the operation count. Since ADIFOR-generated code computes sensitivity analysis with the additional overhead of one function analysis one would expect the ratio of sensitivity cost to function cost for two design variables to be 3. However, the compiler on the SGI™ computer used in this study was able to perform more extensive code optimization to the function analysis portion than it could the sensitivity analysis portion. As a result, the ratio is substantially greater.

The optimization results shown in this work are for design problems involving only two or eight out of fifteen available wing design variables. These present SAADO results are discussed in the context of other SAND approaches at the end of this section. Flow

conditions for the wing optimization examples were $M_\infty = 0.8$ and $\alpha = 1^\circ$.

Two-Design-Variable Problems

Table 1 and Figs. 6 and 7 show results from several optimization problems involving two design variables: the tip chord c_t and the tip setback x_t . Two of these problems are the conventional and SAADO optimizations⁸ using rigid wing analysis. The other problems are optimizations using flexible wing analysis. The difference between the other two sets is definition of the constraints. One set uses the same constraints as the rigid wing optimization problem, denoted as “rigid” constraints in Table 1. That is, minimum total lift, maximum pitching moment and maximum rolling moment. The other uses the previously defined “flexible” constraints; i.e., those constraints that include structural responses.

Figure 6 shows wing planform and surface pressure contours for the initial and optimized designs. The SAADO and conventional “optimized” rigid wings are essentially the same, with the DV differing only in the third significant figure as shown in Table 1. Resulting chordwise pressure distributions are the same, so only results from the SAADO optimization are shown. Similarly, the SAADO and conventional “optimized” flexible wings with either set of constraints show even smaller differences, so only the SAADO result is shown. The shock wave has been weakened substantially in the optimized cases from that on the original wing, as would be expected. This is also evidenced in the chordwise pressure coefficient distributions shown in Fig. 7.

Table 1 compares the values of design variables, objective functions and constraints for 2-DV problems. Due to differences in the analyses caused by differences in meshes, comparisons between optimization problems are made with objective function values normalized by the value obtained from analysis of the initial design. Overall, final designs are very similar between the six problems. Since the problem is dominated by shock strength and there are only two design variables available to change, that is not surprising. The relative computational cost of SAADO optimizations and respective conventional method optimizations is about the same for the two cases with flexible wing analysis as it was for the rigid wing analysis from Ref. 8.

Eight-Design-Variable Problems

Table 2 and Fig. 8 show results from optimization problems using eight design variables as described in Fig. 2. In this case, results for the optimizations using rigid wing analysis with “rigid” constraints are substantially different from those using flexible wing analysis with “flexible” constraints. In particular, constraints on compliance and the difference between lift and weight do not allow the increase in span that was allowed in the rigid case. The results of conventional and SAADO optimizations for the rigid wing analysis were so similar that only the SAADO result is shown. The differences in conventional and SAADO results for the flexible wing are also small; but, the differences in planforms are noticeable enough to be shown in Fig. 8. In all of the optimized results, it is also seen that shock strength has been reduced from that on the original wing.

Computation Cost Comparisons

In view of the consistency of NAND and SAND optimization results, measure of success or failure of the SAADO procedure is then its relative computational expense. Two-design-variable results in Table 1 show the relative cost of conventional and SAADO procedures based on accumulated CPU time. Geometry generator and mesh generator cost were not included for the rigid wing cases because their contributions are minimal relative to cost of the flow solver and flow sensitivity solver as shown in Figs. 4 and 5. For the flexible wing cases, however, those contributions are significant for the coupled system, so they have been included along with the cost for CFD and FEM analyses and their respective gradient analyses. Total cost has been normalized by the cost of one full analysis to the target residual. The SAADO method primarily reduces the cost of the coupled function analysis. In this regard, the SAADO method does show improvement over its conventional counterpart for all methods applied to the two-design variable case as shown in Table 1.

However, for the eight-design-variable flexible wing case (Table 2), the SAADO optimization required more function analysis computations than the conventional counterpart. In all other cases to date, SAADO and conventional optimization processes followed essentially the same path through design space. But for this problem, the SAADO method “took a wrong turn” early in the process and spent more time getting back to the “correct” answer. There are two factors affecting the path through design space: function values and gradient values. That is, steps 1 and 3 affect step 5 in the algorithm described

earlier. Since, for SAADO, neither function values nor gradient values are expected to be well converged until the end of the process, either could introduce the error(s) that caused the “wrong turn”. Previous experience has shown that the gradient values tend to be fairly reliable even at poor convergence levels; therefore, open questions remain concerning these approximations and how problem dependent they might be.

The most computational time is spent computing gradients, even though none of the gradient residual ratios were converged below three orders of magnitude. Early in the respective processes, gradients were not well converged. As the number of design variables is increased, this proportion will grow nearly linearly. The need for faster gradient calculations is apparent. Hou et al.¹ estimated a considerable speed-up attributed to using hand-differentiated adjoint code for 2-D Euler equations. For a single discipline design, such as aerodynamic design, use of adjoint or co-state variables reduces gradient computational times significantly, as shown in a number of the quoted references (See for example 1, 2, 4, 14, 20, 24 and 25.). The SAADO formulation using the discrete adjoint method shown in the Appendix of Ref. 8 is easily extended to coupled aero/structural analysis. It is impractical, however, since the coupled sensitivity analyses would require adjoints for each disciplinary output being transferred, i.e., discretized loads and deflections (See, for example, Ref. 18.). In a tightly or implicitly coupled multidisciplinary analysis, adjoints may prove practical since this system would be analogous to a single discipline.

Further Discussion

Relative cost, based on CPU timing ratios, for SAADO (SAND) versus conventional (NAND) procedures applied to these present small 3-D aerodynamic shape design optimization problems are about seven-tenths for all except the eight-design-variable SAADO case. This range is very similar to that reported for 2-D nonlinear aerodynamic shape design optimization in Refs. 1 and 4, even though many of the computational details differ. The results given in Ref. 1 were for a turbulent transonic flow with shock waves computed using a Navier-Stokes code; a direct differentiation approach (using ADIFOR) was used for the sensitivity analysis. The results reported in Ref. 4 were for a compressible flow without shock waves computed using a nonlinear potential flow code; an adjoint approach was used for the sensitivity analysis. Since these two optimization problems were also not the same, then, no timing comparison between these adjoint and

direct differentiation solution approaches would be meaningful. As indicated earlier, an expected speed-up was estimated in Ref. 1 for using an adjoint approach instead of direct differentiation.

Ghattas and Bark²⁵ recently reported 2-D and 3-D results for optimal control of steady incompressible Navier-Stokes flow which demonstrate an order-of-magnitude reduction of CPU time for a SAND approach versus a NAND approach. These results were obtained using reduced Hessian SQP methods that avoid converging the flow equations at each optimization iteration. The relationship of these methods with respect to other optimization techniques is also discussed in Ref. 25.

Several other SAND-like methods for simultaneous analysis and design are summarized and discussed by Ta'asan.³⁹ These methods are called "One-Shot" and "Pseudo-Time" and have been applied to aerodynamic shape design problems at several fidelities of CFD approximation, as noted in Ref. 39. These techniques have obtained an aerodynamic design in the equivalent of several analysis CPU times for some sample problems.

CONCLUDING REMARKS

This study has introduced an implementation of the SAADO technique for a simple, isolated, flexible wing. Initial results indicate that SAADO

1. is feasible under dual simultaneity (i.e. simultaneity not only with respect to analysis and design optimization, but also simultaneity with respect to flexible wing aero-structural interaction)
2. finds the same local minimum as a conventional technique
3. can be computationally more efficient than a conventional gradient-based optimization technique; however, the relative efficiency may be dependent on the optimization problem
4. requires few modifications to the analysis and sensitivity analysis codes involved.

Perhaps improvements to this SAADO procedure or its implementation can be made with respect to gradient-approximation and line-search techniques.

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Table 1. Comparison of two-design-variable results.

	Analysis @ initial shape		Optimization Results					
			Rigid (Ref. 8)		Flexible w/ 'rigid' constraints		Flexible w/ 'flexible' constraints	
	Rigid (Ref. 8)	Flexible	Conv	SAADO	Conv	SAADO	Conv	SAADO
F_0	-8.43	-7.15						
F/F_0	1	1	1.466	1.465	1.26	1.26	1.397	1.396
g_1	-0.0822	-0.0302	-0.0003	0.0003	0.0002	0	0	-0.0023
g_2	-0.9276	-0.8882	-0.9928	-0.9931	-0.9836	-0.9843	-0.9191	-0.9246
g_3	-0.532	-0.2647	-0.5671	-0.5612	-0.5752	-0.5755	-0.0968	-0.0938
dv 1 (c_t)	1	1	1.07	1.04	1.11	1.11	1.13	1.14
dv 2 (x_t)	1	1	2.03	1.97	1.88	1.89	1.94	1.95
Cost	1	1	20.6	15.1	31.2	19.1	36.2	22.2

Table 2. Comparison of 8-design-variable optimization results.

	Analysis @ initial shape		Optimization Results			
			Rigid		Flexible	
	Rigid	Flexible	Conv	SAADO	Conv	SAADO
F_0	-8.43	-7.15				
F/F_0	1	1	3.48	3.48	2.83	2.78
g_1	-0.0822	-0.0302	-2.48	-2.49	-0.464	-0.225
g_2	-0.928	-0.888	0.0009	-0.0004	-0.0003	-0.0003
g_3	-0.532	-0.265	0.0002	0.0005	-0.0013	-0.0007
dv 1 (c_t)	1	1	0.844	0.836	0.605	0.577
dv 2 (x_t)	1	1	3	3	2.29	1.88
dv 3 (b)	1	1	1.5	1.5	0.851	0.878
dv 4 (t)	1	1	0.625	0.62	1.43	1.45
dv 5 (t_r)	1	1	0.5	0.5	0.5	0.5
dv 6 (t_i)	1	1	0.941	0.565	0.29	0.389
dv 7 (z_r)	1	1	1.13	1.14	1.11	1.09
dv 8 (x_{zr})	1	1	0.936	0.944	0.651	0.6
Cost	1	1	152	128	120	117

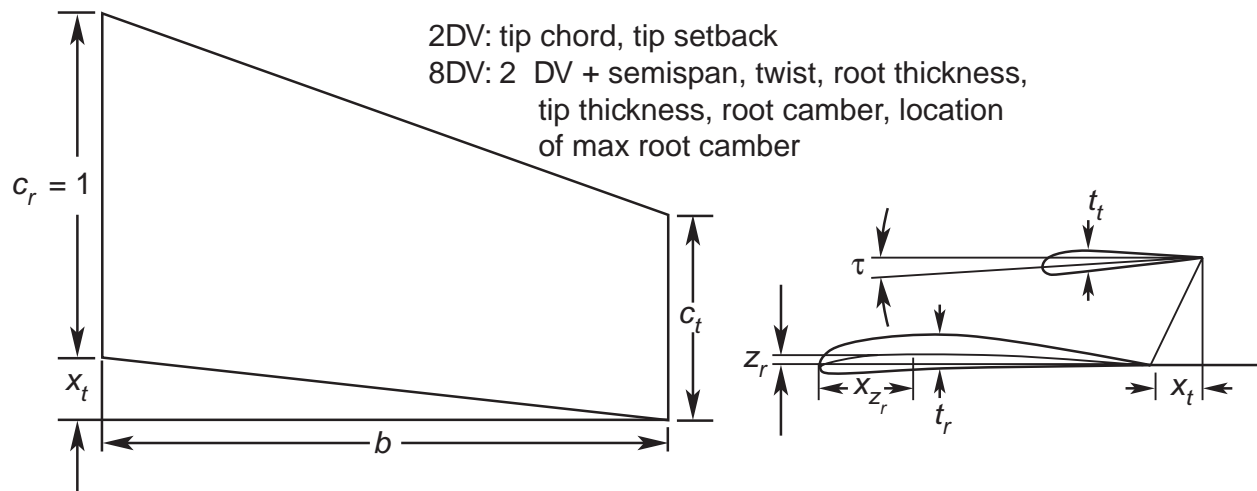


Figure 1. Description of semispan wing parameterization.

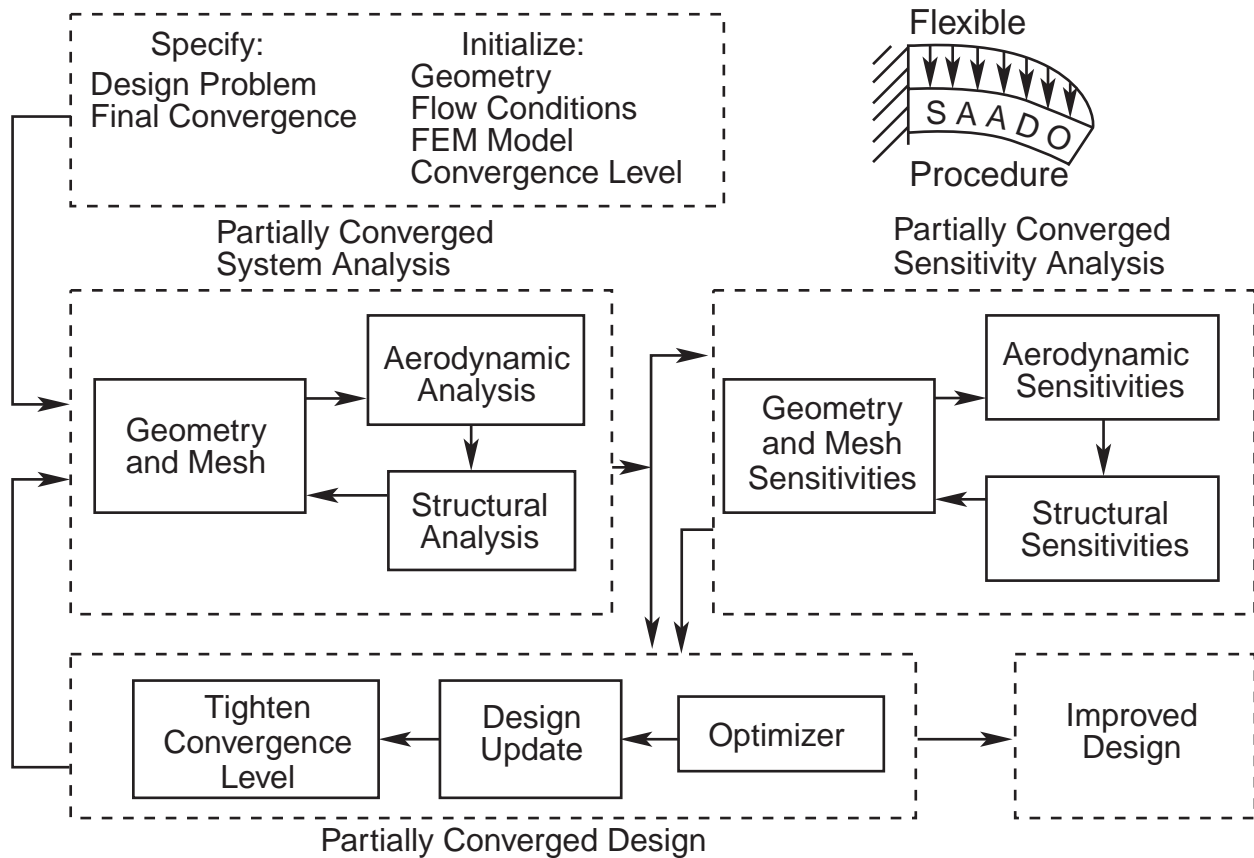


Figure 2. Diagram of flexible wing SAADO procedure.

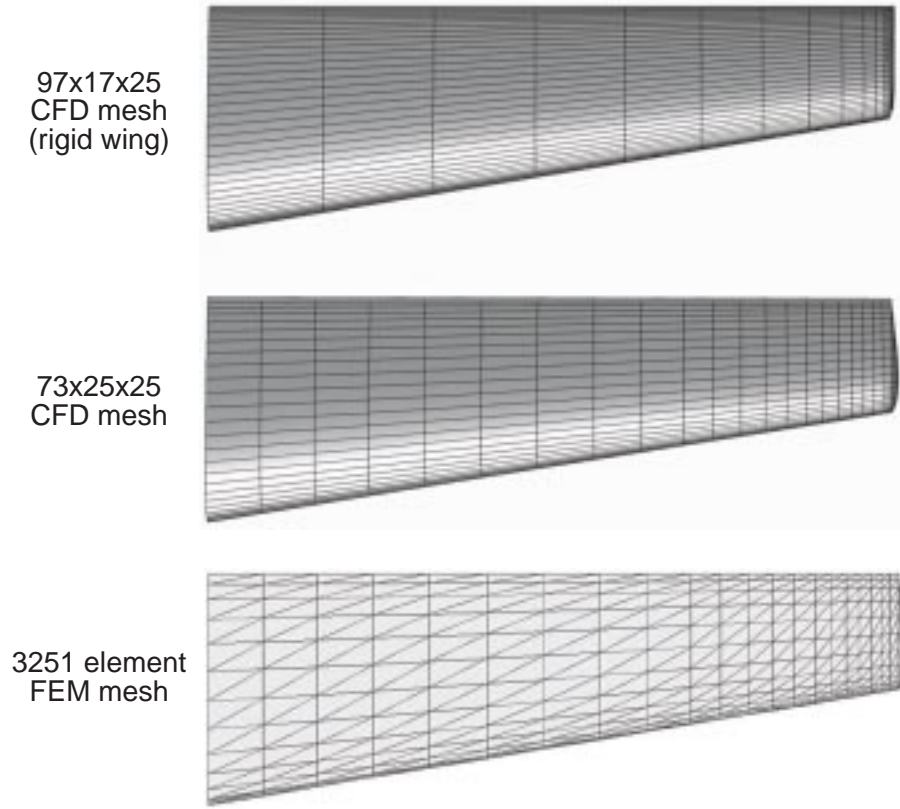


Figure 3. Computational meshes for rigid wing analysis and coupled flexible wing analysis.

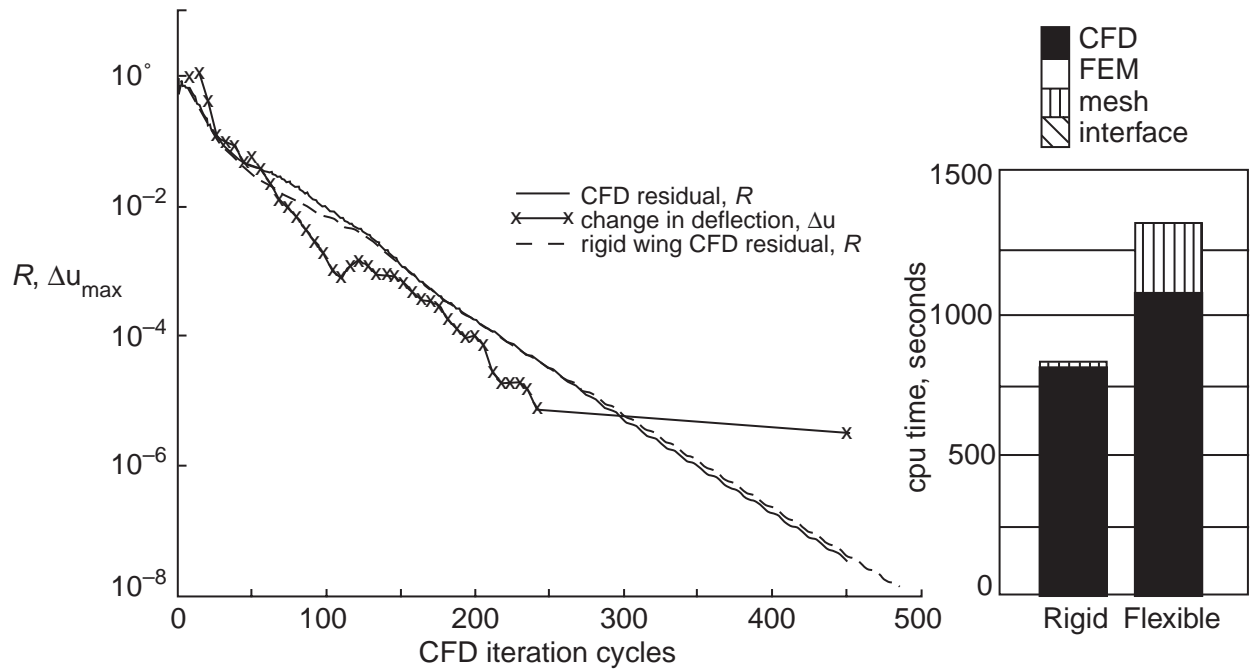


Figure 4. Effect of aerodynamic/structural coupling on function analysis convergence, $M_\infty = 0.8$, $\alpha = 1^\circ$.

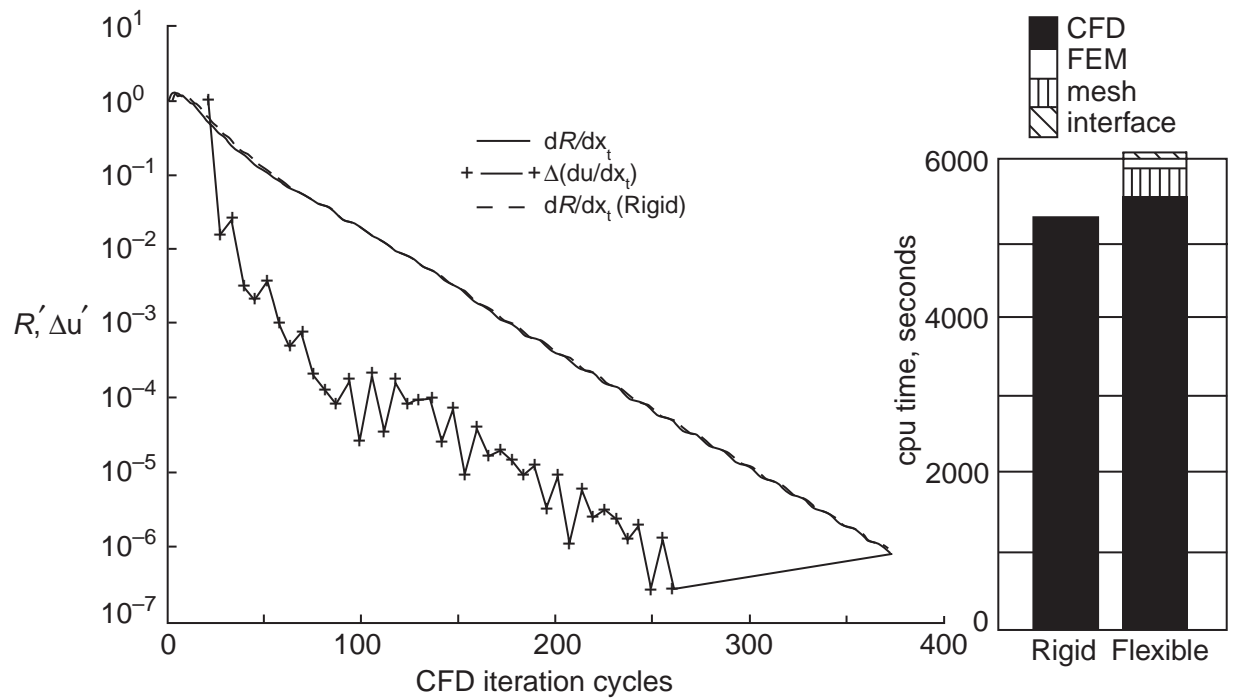


Figure 5. Effect of aerodynamic/structural coupling on sensitivity analysis convergence.

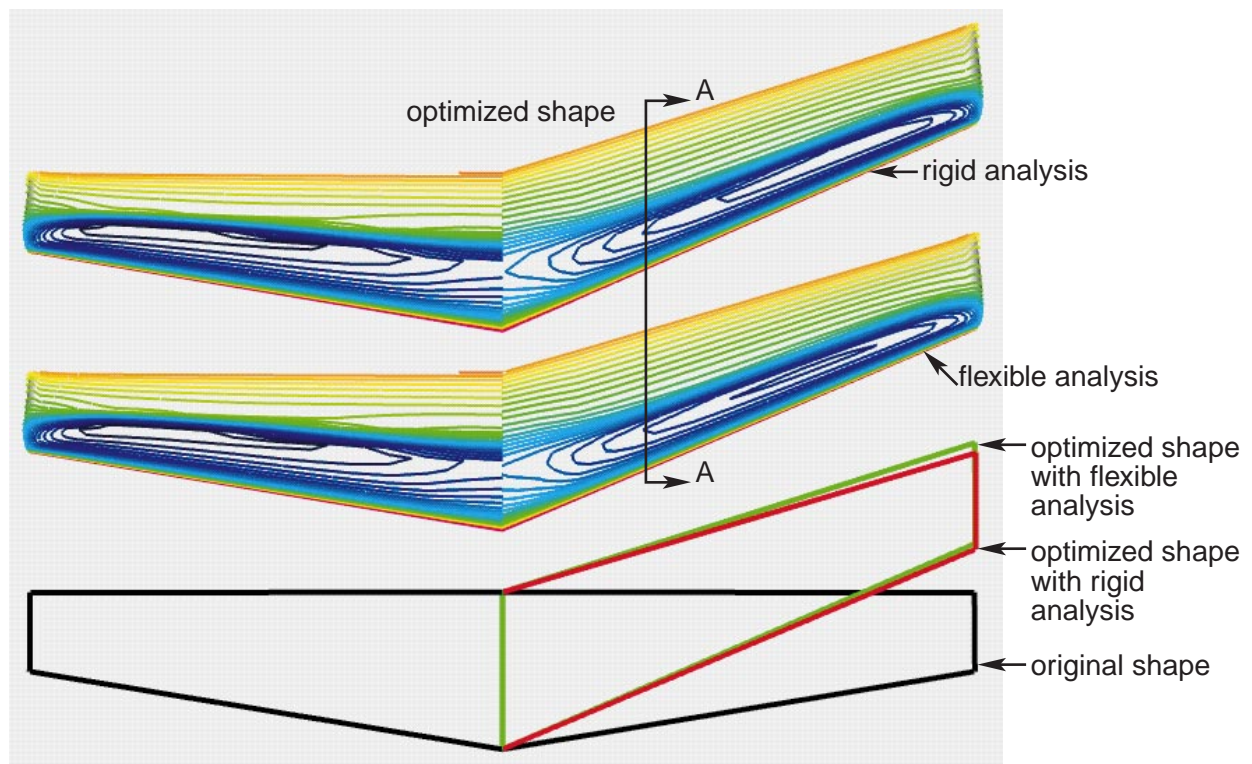


Figure 6. Comparison of planform shapes and surface pressure contours for two-design-variable cases, $M_\infty = 0.8$, $\alpha = 1^\circ$.

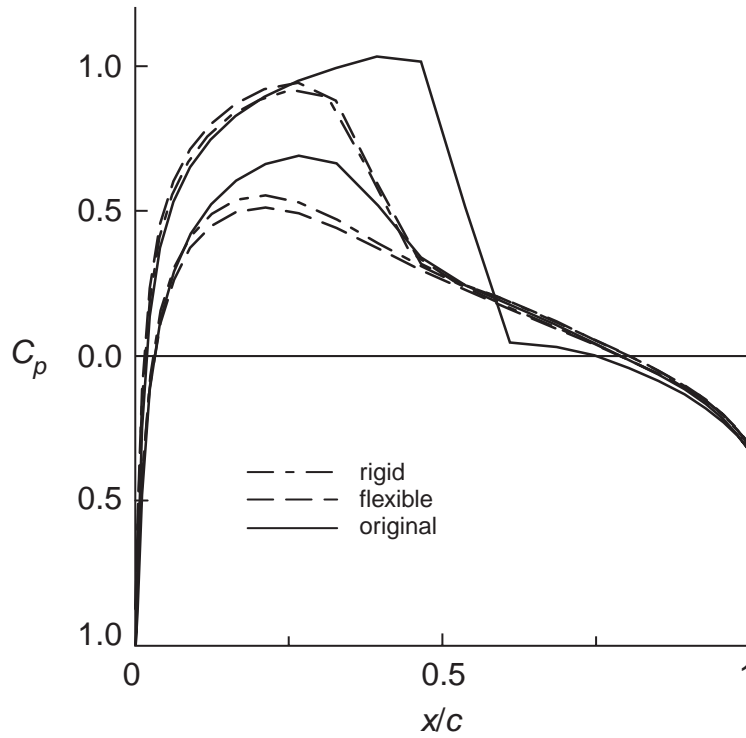


Figure 7. Comparison of chordwise pressure coefficient distributions at section A-A for two-design-variable cases, $M_\infty = 0.8$, $\alpha = 1^\circ$.

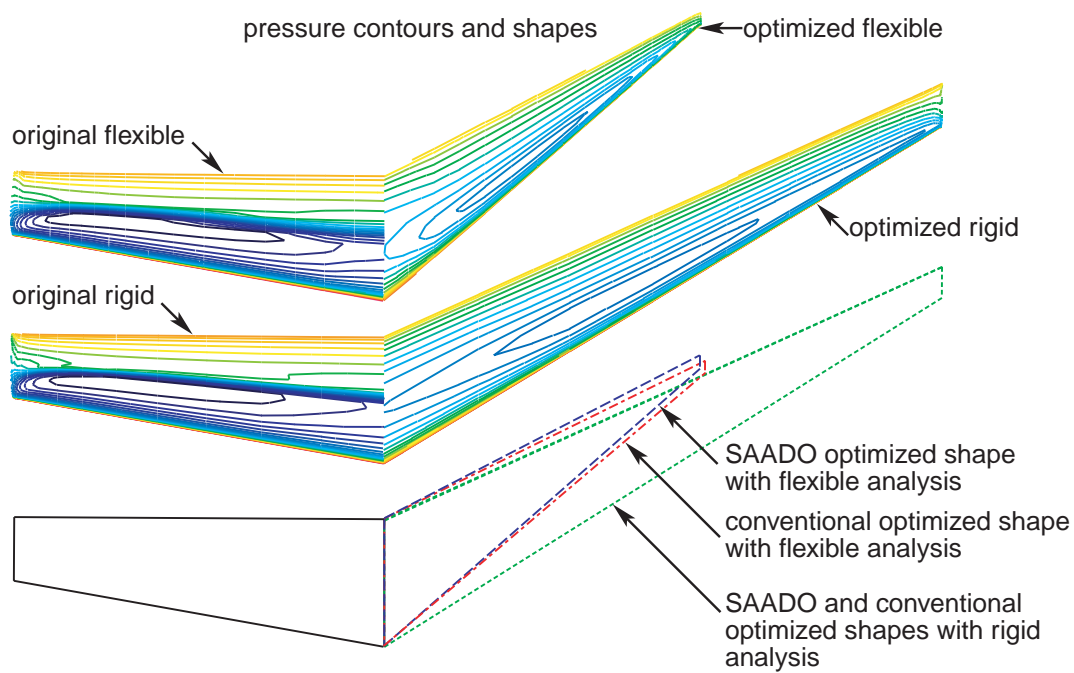


Figure 8. Comparison of planform shapes and surface pressure contours for eight-design-variable cases, $M_\infty = 0.8$, $\alpha = 1^\circ$.